

Express each of the following in the form  $r \sin(x+a)$ , where  $r>0$  and  $0<a<2\pi$ .

(a)  $5\cos x + 12\sin x$

(b)  $12\cos x + 5\sin x$

Solution:

$$5^2 + 12^2 = 13^2$$

$$\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = 1$$

(a)  $\sin a = \frac{5}{13}, \cos a = \frac{12}{13}$

$$\sin^2 a + \cos^2 a = 1$$

$$5\cos x + 12\sin x = 13\left(\frac{5}{13}\cos x + \frac{12}{13}\sin x\right) = 13(\sin a \cos x + \cos a \sin x) = 13\sin(x+a)$$

(b)  $\sin a = \frac{12}{13}, \cos a = \frac{5}{13}$

$$12\cos x + 5\sin x = 13\left(\frac{12}{13}\cos x + \frac{5}{13}\sin x\right) = 13(\sin a \cos x + \cos a \sin x) = 13\sin(x+a)$$

*Answer: (a)  $13\sin(x+a)$ , (b)  $13\sin(x+a)$ .*