Determine whether there is a function $\varphi(x,y,z)$ such that $F = grad\big(\varphi(x)\big)$, where

1) $F = (xz - y)i + (x^2 + z^3)j + (3xz^2 - xy)k$

Solution: Set initial point (0,0,0).

So

$$\varphi(x,y,z) = \int_0^x P(t,0,0)dt + \int_0^y Q(x,t,0)dt + \int_0^z R(x,y,t)dt\ ,$$

where $P(x,y,z) = (xz - y), Q(x,y,z) = (x^2 + z^3), R(x,y,z) = (3xz^2 - xy)$.

$$\varphi(x,y,z) = \int_0^x (0-0)dt + \int_0^y (x^2)dt + \int_0^z (3xt^2 - xy)dt = x^2t\,\Big|_0^y + (xt^3 - xyt)\,\Big|_0^z = x^2y + xz^3 - xyz.$$

Answer: $\varphi(x,y,z) = x^2y + xz^3 - xyz$

2) $F = 2xe^{-y}i + (\cos z - x^2e^{-y})j - ysin\, z\, k$

Solution: Set initial point (0,0,0).

So

$$\varphi(x,y,z) = \int_0^x P(t,0,0)dt + \int_0^y Q(x,t,0)dt + \int_0^z R(x,y,t)dt\ ,$$

where $P(x,y,z) = 2xe^{-y}, Q(x,y,z) = (\cos z - x^2e^{-y})j\ , R(x,y,z) = ysin\, z$.

$$\varphi(x,y,z) = \int_0^x 2tdt + \int_0^y (1 - x^2e^{-t})dt + \int_0^z ysintdt = t^2 \Big|_0^x + (t + x^2e^{-t}) \Big|_0^y - ycos\, t \Big|_0^z = x^2 + y + x^2e^{-y} - x^2 - ycos\, z + y = x^2e^{-y} + 2y - y\cos z\,.$$

Answer: $\varphi(x,y,z) = x^2e^{-y} + 2y - y\cos z$