Conditions

Verify Green's Theorem in the plane for integral_c [(x^2-xy^3)dx +(y^2-2xy)dy] where C is a square with vertices at (0,0), (2,0), (2,2), (0,2).

Solution

As we know, the Green's Theorem claims:

Let C be a positively oriented, piecewise smooth, simple closed curve in a plane, and let D be the region bounded by C. If L and M are functions of (x, y) defined on an open region containing D and have continuous partial derivatives there, then:

$$\oint_C (L\,\mathrm{d}x + M\,\mathrm{d}y) = \iint_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right)\,\mathrm{d}x\,\mathrm{d}y.$$

Let's check the conditions of this theorem to check whether it works for our case.

As the C is a square with vertices at (0,0), (2,0), (2,2), (0,2), then this is positively oriented, piecewise smooth (because each side of this square could be represented as a linear function on a plane), simple closed curve. So, the conditions for C is completed.

Let's check the partial derivatives for functions:

$L(x, y) = x^2 - xy^3$

$M(x, y) = y^2 - 2xy$

As these functions are polynomials, then each their derivative is a continuous function. And these functions are defined in all R^2, so they are defined on each open region containing D.

<u>That's why the Green's Theorem is verified for our example.</u>