

Conditions

Solve the set of linear equations by the matrix method : $a+3b+2c=3$, $2a-b-3c= -8$, $5a+2b+c=9$.
Solve for a and b

Solution

$$\begin{cases} a + 3b + 2c = 3 \\ 2a - b - 3c = -8 \\ 5a + 2b + c = 9 \end{cases}$$

Let's construct the coefficient matrix of this system:

$$\begin{pmatrix} 1 & 3 & 2 & 3 \\ 2 & -1 & -3 & -8 \\ 5 & 2 & 1 & 9 \end{pmatrix}$$

Now reduce the matrix to echelon form:

$$\begin{pmatrix} 1 & 3 & 2 & 3 \\ 2 & -1 & -3 & -8 \\ 5 & 2 & 1 & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & -7 & -7 & -14 \\ 0 & -13 & -9 & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & -1 & -1 & -2 \\ 0 & -13 & -9 & -6 \end{pmatrix} \sim \\ \sim \begin{pmatrix} 1 & 3 & 2 & 3 \\ 0 & -1 & -1 & -2 \\ 0 & 0 & 4 & 20 \end{pmatrix}$$

We've got a system:

$$\begin{cases} a + 3b + 2c = 3 \\ -b - c = -2 \\ 4c = 20 \end{cases}$$

Now we can see, that

$$c = 5$$

$$b = 2 - c = -3$$

$$a = 3 - 3b - 2c = 3 + 9 - 10 = 2$$

Answer:

$$a = 2$$

$$b = -3$$

$$c = 5$$

