

Conditions

prove that integral of $\sin^2 w dw/w^2$ having limits 0 to infinite = $\pi/2$??

Solution

Consider the integral:

$$\int_0^{\infty} \frac{\sin^2 w dw}{w^2}$$

As we know, this integral cannot be represented by using elementary functions, that's why in math appeared the definition of Sine integral:

$$\text{Si}(x) = \int_0^x \frac{\sin t}{t} dt$$

One of the properties of this integral is:

$$\lim_{x \rightarrow +\infty} \text{Si } x = \frac{\pi}{2},$$

It's known, that

$$\int \frac{\sin^2 w dw}{w^2} = \text{Si}(2w) - \frac{\sin^2 w}{w} + c$$

$$\int_0^{\infty} \frac{\sin^2 w dw}{w^2} = \lim_{x \rightarrow \infty} \int_0^x \frac{\sin^2 w dw}{w^2} = \lim_{x \rightarrow \infty} \text{Si}(2x) - 0 - 0 + 0 = \frac{\pi}{2}$$