1. True or false:

- If f is a linear function, then f has an inverse? FALSE
- Is it f(x)=3 is a linear function? TRUE

2. Let f(x)= definite integral of the square root (1+t^3) dt with lower and upper limit are 3 and x respectively.

(a) argue that f has an inverse function
(b) find f^-1(0)

Solution:

(a) Since $f(x) = \int_3^x \sqrt{1+t^3}dt$ is an area, one x will produce only one area f(x).
Since x >0, one area has only one corresponding x.
Thus, f(x) has an inverse function.

(b) 1. Find f(3):

$$f(3) = \int_3^3 \sqrt{1+t^3}dt = F(3) - F(3) = 0$$

2. Find f'(x) and f'(3):

$$f'(x) = \sqrt{1+x^3} \cdot 1 = \sqrt{1+x^3}$$

$$f'(3) = \sqrt{1+3^3} = \sqrt{28}$$

3. Let the inverse of f(x) will be g(x), then

$$f(g(x)) = x$$

Take derivatives on both sides.

$$f'(g(x)) \cdot g'(x) = 1 \quad (1)$$

Set x =0

$$f'(g(0)) \cdot g'(0) = 1$$

Also,

$$g(f(x)) = x$$

Thus,

$$g(f(3)) = 3$$

That is,

$$g(0) = 3$$

Take it back to (1)

$$f'(3) \cdot g'(0) = 1$$

$$\sqrt{28} \cdot g'(0) = 1$$

$$g'(0) = \frac{1}{\sqrt{28}}$$

That is $f^{-1}(0) = \frac{1}{\sqrt{28}}$