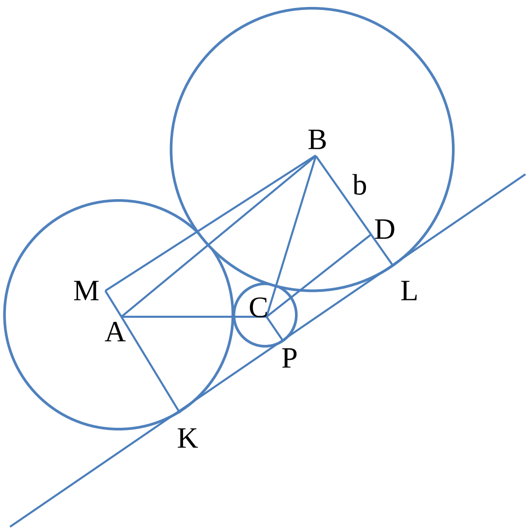
two circles with radii a and b respectively touch each other externally and a smaller circle with radius c touch both the circles and also the common tangent to the two circles, then prove that:

{(1/a)^0.5} +{(1/b)^0.5} ={(1/c)^0.5}

Solution



Circles are touch each other externally, thus:

$$AB = a + b$$

$$AC = a + c$$

$$BC = c + b$$

From right triangle $\Delta CDB, where\ \angle D = 90\ °$, using Pythagorean theorem:

$$CD = \sqrt{(BC)^2 - (BD)^2}$$

As:

$$BD = BL - DL = b - c$$

So:

$$CD = \sqrt{(c+b)^2 - (c-b)^2} = \sqrt{c^2 + 2cb + b^2 - c^2 + 2cb - b^2} = \sqrt{4cb}$$

From rectangle CDLP:

$$CD = LP$$

Thus:

$$LP = 2\sqrt{cb}$$

Similarly :

$$PK = 2\sqrt{ca}$$

So:

$$LK = LP + PK = \ 2\sqrt{c}\left(\sqrt{b} + \sqrt{a}\right)$$

Similarly, from right triangle $\Delta AMB, where\ \angle M = 90\ °$, using Pythagorean theorem:

$$BM = \sqrt{(b+a)^2 - (b-a)^2} = 2\sqrt{ab}$$

From rectangle KMBL:

$$BM = LK$$

Thus:

$$2\sqrt{ab} = \ 2\sqrt{c}\left(\sqrt{b} + \sqrt{a}\right)$$

Dividing by $\frac{\sqrt{c}}{2\sqrt{ab}}$ gives:

$$\boxed{\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}}$$

Or :

$$\left(\frac{1}{a}\right)^{0.5} + \left(\frac{1}{b}\right)^{0.5} = \left(\frac{1}{c}\right)^{0.5}$$