**Question 1.** *In* $V_3(\mathbb{R})$ *find* $\dim(A+B)$ *and* $\dim(A \cap B)$ *where* $A$ *is the subspace spanned by* $(1,1,1)$ *and* $B$ *is the subspace spanned by* $(-1,-1,-1)$.

*Solution.* Note that the vectors $(1,1,1)$ and $(-1,-1,-1)$ are linearly dependent, namely, $(-1,-1,-1) = -(1,1,1)$. Therefore, the 1-dimensional subspaces spanned by these vectors coincide. More precisely, each vector of $A$ has the form $\alpha(1,1,1)$ for some $\alpha \in \mathbb{R}$ and this vector also belongs to $B$, because $\alpha(1,1,1) = -\alpha(-1,-1,-1)$. And conversely, each vector of $B$ lies in $A$ by the similar reason. Thus, $A = B$ and so

$$A + B = A + A = A,$$
$$A \cap B = A \cap A = A.$$

Hence, $\dim(A+B) = \dim(A \cap B) = \dim A = 1$.
*Answer*: $\dim(A+B) = \dim(A \cap B) = 1$. □