Question 1. *Find the dimension of the subspace spanned by the vectors* e_1, e_2, e_3 *in* $V_4(\mathbb{R})$.

Solution. Recall that

$$\begin{aligned} e_1 &= (1, 0, 0, 0), \\ e_2 &= (0, 1, 0, 0), \\ e_3 &= (0, 0, 1, 0). \end{aligned}$$

Prove that e_1, e_2, e_3 are linearly independent in over $\mathbb{R}$. Indeed, for any $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$ we have

$$\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 = (\alpha_1, \alpha_2, \alpha_3, 0).$$

So, if $\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 = (0, 0, 0, 0)$, then immediately $\alpha_1 = \alpha_2 = \alpha_3 = 0$. Since these vectors are linearly independent, they form a basis of the space spanned by them. Thus, the dimension of this space equals the number of the vectors e_1, e_2, e_3, i. e. it is 3.
Answer: 3. $\square$