Conditions

Find the probability of obtaining 6 or more hads when an ordinary coin is tossed 10 times. Calculate also the approximate probability of obtaining more than 90 heads when the coin is tossed 100 times.

Solution

We can use the Bernoulli's formula to calculate these probabilities. This formula is being used, when we want to find the probability of event which outcome in some series of independent tests and where we know the probability of this event outcome in such test.

$$P = C_n^k p^k q^{n-k}$$

Let's find the probability of obtaining 6,7,8,9 and 10 hads when an ordinary coin is tossed 10 times:

$$n = 10, k = 6, p = q = \frac{1}{2}$$

$$P = \frac{10!}{4!\,6!}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 = \frac{10\cdot 9\cdot 8\cdot 7}{2\cdot 3\cdot 4}\left(\frac{1}{2}\right)^{10} = 0{,}205078125$$

Analogically:

$$n = 10, k = 7, p = q = \frac{1}{2}$$

$$P = 0.1171875$$

$$n = 10, k = 8, p = q = \frac{1}{2}$$

$$P = 0.0439453125$$

$$n = 10, k = 9, p = q = \frac{1}{2}$$

$$P = 0.009765625$$

$$n = 10, k = 10, p = q = \frac{1}{2}$$

$$P = 0.0009765625$$

And the of obtaining 6 or more hads when an ordinary coin is tossed 10 times is the sum of these probabilities:

$$P = 0{,}376953125$$

It’s approximately 37.8% chance.

Let’s find the probability of obtaining 90-100 hads when an ordinary coin is tossed 100 times. It’s the sum of 10 probabilities analogically for previous calculations:

$$n = 100, k = 90, p = q = \frac{1}{2}$$

$$P = \frac{100!}{10!\,90!}\left(\frac{1}{2}\right)^{90}\left(\frac{1}{2}\right)^{10}$$

And so on and so forth.

$$P = 1.53165 \cdot 10^{-17}$$