

Conditions

Integrate $(\tan 2x + \cot 2x)^2 dx$

Solution

1. Transform tan and cot to the sin-cos terms: $\tan = \frac{\sin}{\cos}$ $\cot = \frac{\cos}{\sin}$ and sum.
2. Using the property that $\sin^2 2x + \cos^2 2x = 1$ and $\sin 2x \cos 2x = \frac{1}{2} \sin 4x$
3. Transform $\frac{1}{\sin^2 4x}$ into a csc form
4. Using the table integral for $\csc^2 4x$

$$\begin{aligned} \int (\tan 2x + \cot 2x)^2 dx &= \int \left(\frac{\sin^2 2x + \cos^2 2x}{\sin^2 2x \cos^2 2x} \right)^2 dx = \int \frac{4}{\sin^4 4x} dx = 4 \int \csc^2 4x dx = \\ &= 4 \left(-\frac{\cot 4x}{4} + c \right) = -\cot 4x + c \end{aligned}$$