Conditions

5) let A=(2 1 0)

(1 1 4) find all the minors, co factors and the inverse of the given matrices

(-3 2 5)

Solution

Consider matrix:

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 4 \\ -3 & 2 & 5 \end{pmatrix}$$

Let's find all minors:

$$M_{11} = \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} = 5 - 8 = -3$$

$$M_{12} = \begin{vmatrix} 1 & 4 \\ -3 & 5 \end{vmatrix} = 5 + 12 = 17$$

$$M_{13} = \begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix} = 2 + 3 = 5$$

$$M_{21} = \begin{vmatrix} 1 & 0 \\ 2 & 5 \end{vmatrix} = 5 - 0 = 5$$

$$M_{22} = \begin{vmatrix} 2 & 0 \\ -3 & 5 \end{vmatrix} = 10 - 0 = 10$$

$$M_{23} = \begin{vmatrix} 2 & 1 \\ -3 & 5 \end{vmatrix} = 10 + 3 = 13$$

$$M_{31} = \begin{vmatrix} 1 & 0 \\ 1 & 4 \end{vmatrix} = 4 - 0 = 4$$

$$M_{32} = \begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix} = 8 - 0 = 8$$

$$M_{33} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2 - 1 = 1$$

Now it's time for cofactors. As we know:

$$A_{ij} = (-1)^{i+j} M_{ij}$$

That's why:

$$A_{11} = -3$$

$$A_{12} = -17$$

$$A_{13} = 5$$

$$A_{21} = -5$$

$$A_{22} = 10$$

$$A_{23} = -13$$

$$A_{31} = 4$$

$$A_{32} = -8$$

$$A_{33} = 1$$

Now let's construct the inverse of matrix A. First of all we must construct the cofactor matrix:

$$A_C = \begin{pmatrix} -3 & -17 & 5 \\ -5 & 10 & -13 \\ 4 & -8 & 1 \end{pmatrix}$$

Then transpose it:

$$A_C{}^T = \begin{pmatrix} -3 & -5 & 4 \\ -17 & 10 & -8 \\ 5 & -13 & 1 \end{pmatrix}$$

And in the end, divide this on $\det A$:

$$A^{-1} = \frac{1}{\det A} A_C{}^T$$

$$\det A = \det \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 4 \\ -3 & 2 & 5 \end{pmatrix} = 10 - 12 - 0 - 0 - 16 - 5 = -23$$

$$A^{-1} = \frac{1}{\det A} A_C{}^T = \begin{pmatrix} \frac{3}{23} & \frac{5}{23} & -\frac{4}{23} \\ \frac{17}{23} & -\frac{10}{23} & -\frac{8}{23} \\ -\frac{5}{23} & \frac{13}{23} & -\frac{1}{23} \end{pmatrix}$$