Task:

A car is traveling 60 mph A bike is seen 100 yards away going 20 mph. First case is the bike is traveling in the same direction as the car; second case is that the bike is moving towards the car. The question is what is the difference in time of "closure"? (i.e., when do they arrive at the same point? What is the point(time and distance) where they would collide? Also HOW do you solve this. And is the answer for other speeds and distances linear?

Solution:

1. Due to Newton's second law:
   $\sum F = ma$
   $a = \frac{\sum F}{m}$
   $a = \dot{v}$
   $v = \int a\, dt = at + v_0$
   $t = \left(\frac{v - v_0}{a}\right)$
   $v = \dot{s}$
   $s = \int v\, dt = \int (at + v_0)\, dt = \frac{at^2}{2} + v_0 t + s_0 = \frac{a\left(\frac{v - v_0}{a}\right)^2}{2} + v_0\left(\frac{v - v_0}{a}\right) + s_0 =$
   $= \frac{v^2 - 2vv_0 + {v_0}^2}{2a} + \frac{vv_0 - {v_0}^2}{a} + s_0 = \frac{v^2 - {v_0}^2}{2a} + s_0$
   Given:
   $v_{0_{car}} = 60\frac{mile}{h} \cdot 1609.344\frac{m}{mile} = 60\frac{mile}{3600\ s} \cdot 1609.344\frac{m}{mile} = 26.8224\frac{m}{s}$
   $s_{0_{car}} = 0\ m$
   $s_{0_{bike}} = 100\ yard \cdot 0.9144\frac{m}{yard} = 91.44\ m$
   $v_{0_{bike}} = 20\frac{mile}{h} \cdot 1609.344\frac{m}{mile} = 20\frac{mile}{3600\ s} \cdot 1609.344\frac{m}{mile} = 8.90408\frac{m}{s}$
   $a_{\text{bike}} = a_{car} = 0\frac{m}{s^2}$

   $s = \frac{a_{car}t^2}{2} + v_{0_{car}}t + s_{0_{car}} = 26.8224\frac{m}{s} \cdot t$
   $s = \frac{a_{bike}t^2}{2} + v_{0_{bike}}t + s_{0_{bike}} = 8.90408\frac{m}{s} \cdot t + 91.44\ m$
   Solve the system of equations:

Solve the following system:

$$\begin{cases} s = 26.8224\, t \\ s = 8.90408\, t + 91.44 \end{cases}$$

Perform a substitution.

Substitute $s = 26.8224\, t$ into the second equation:

$$\begin{cases} s = 26.8224\, t \\ \boxed{26.8224\, t = 8.90408\, t + 91.44} \end{cases}$$

Choose an equation and a variable to solve for.

In the second equation, look to solve for t:

$$\begin{cases} s = 26.8224\, t \\ \boxed{26.8224\, t = 8.90408\, t + 91.44} \end{cases}$$

Isolate t to the left hand side.

Subtract $8.90408\, t$ from both sides:

$$\begin{cases} s = 26.8224\, t \\ \boxed{17.9183\, t = 91.44} \end{cases}$$

Solve for t.

Divide both sides by 17.9183:

$$\begin{cases} s = 26.8224\, t \\ \boxed{t = 5.10316} \end{cases}$$

Perform a back substitution.

Substitute $t = 5.10316$ into the first equation:

Answer:

$$\begin{cases} \boxed{s = 136.879} \\ t = 5.10316 \end{cases}$$

2. Given:

$v_{0_{car}} = 60 \cdot 1609.344 \frac{m}{3600\ s} = 26.8224 \frac{m}{s}$

$s_{0_{car}} = 0\ m$

$s_{0_{bike}} = 100 \cdot 0.9144\ m = 91.44\ m$

$v_{0_{bike}} = -20 \cdot 1609.344 \frac{m}{3600\ s} = -8.90408 \frac{m}{s}$

$a_{\text{bike}} = a_{car} = 0 \frac{m}{s^2}$

$s = \frac{a_{car} t^2}{2} + v_{0_{car}} t + s_{0_{car}} = 26.8224 \frac{m}{s} \cdot t$

$s = \frac{a_{bike} t^2}{2} + v_{0_{bike}} t + s_{0_{bike}} = -8.90408 \frac{m}{s} \cdot t + 91.44\ m$

Solve the system of equations:

Solve the following system:

$$\begin{cases} s = 26.8224\,t \\ s = 91.44 - 8.90408\,t \end{cases}$$

Perform a substitution.

Substitute $s = 91.44 - 8.90408\,t$ into the first equation:

$$\begin{cases} \boxed{91.44 - 8.90408\,t = 26.8224\,t} \\ s = 91.44 - 8.90408\,t \end{cases}$$

Choose an equation and a variable to solve for.

In the first equation, look to solve for t:

$$\begin{cases} \boxed{91.44 - 8.90408\,t = 26.8224\,t} \\ s = 91.44 - 8.90408\,t \end{cases}$$

Isolate t to the left hand side.

Subtract $26.8224\,t + 91.44$ from both sides:

$$\begin{cases} \boxed{-35.7265\,t = -91.44} \\ s = 91.44 - 8.90408\,t \end{cases}$$

Solve for t.

Divide both sides by -35.7265:

$$\begin{cases} \boxed{t = 2.55945} \\ s = 91.44 - 8.90408\,t \end{cases}$$

Perform a back substitution.

Substitute $t = 2.55945$ into the second equation:

$$\begin{cases} t = 2.55945 \\ \boxed{s = 68.6505} \end{cases}$$

Sort results.

Collect results in alphabetical order:

Answer:

$$\begin{cases} s = 68.6505 \\ t = 2.55945 \end{cases}$$

Answer:

1. $s = 136.879\ m$
   $t = 5.103\ s$

2. $s = 68.6505\ m$
   $t = 2.559\ s$

3. The answer will remain linear as long as accelerations equal zero