Prove that the line joining the mid points of the diagonals of a trapezium is parallel to the parallel sides and is equal to the half of their difference.


Since, $\mathrm{AB} \| \mathrm{DC}$ and transversal AC cuts then at A and C respectively.
$\therefore \angle 1=\angle 2$ (Alternate angles)
Now, In $\triangle \mathrm{APR}$ and $\triangle \mathrm{DPC}$,
$\angle 1=\angle 2$
$\mathrm{AP}=\mathrm{CP}(\mathrm{P}$ is mid point of AC$)$
$\angle 3=\angle 4$ (Vertically opposite angles)
$\therefore \triangle \mathrm{APR} \cong \triangle \mathrm{DPC}$
$\Rightarrow \mathrm{AR}=\mathrm{DC}$ and $\mathrm{PR}=\mathrm{DP}$

In $\triangle \mathrm{DRB}, \mathrm{P}$ and Q are the mid-points of sides DR and DB respectively.
$\therefore \mathrm{PQ} \| \mathrm{RB}$
$\Rightarrow \mathrm{PQ} \| \mathrm{AB}$
$\Rightarrow \mathrm{PQ} \| \mathrm{AB}$ and DC
Again P and Q are the mid-points of sides DR and DB respectively.
$\therefore \mathrm{PQ}=\frac{1}{2} \mathrm{RB}$
$\Rightarrow P Q=\frac{1}{2}(A B-A R)$
$\Rightarrow P Q=\frac{1}{2}(A B-D C)$

