Question

Let takes that we have two vectors: $x=(x_1,y_1)$ and $y=(x_2,y_2)$. These vectors are perpendicular, so we have: $\cos\alpha=\frac{x_1x_2+y_1y_2}{\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}}=0\Rightarrow x_1x_2+y_1y_2=0$ and as they have equal length, then $\sqrt{x_1^2+y_1^2}=\sqrt{x_2^2+y_2^2}\Rightarrow x_1^2+y_1^2=x_2^2+y_2^2$.

Then their sum and difference will be equal: $x+y=(x_1+x_2,y_1+y_2)$ and $x-y=(x_1-x_2,y_1-y_2)$. And in this case we have:

$$\cos\beta=\frac{(x_1+x_2)\cdot(x_1-x_2)+(y_1+y_2)\cdot(y_1-y_2)}{\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}}=\frac{x_1^2-x_2^2+y_1^2-y_2^2}{\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}}=$$

$$=\frac{(x_1^2+y_1^2)-(x_2^2+y_2^2)}{\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}}=\left|x_1^2+y_1^2=x_2^2+y_2^2\right|=0\Rightarrow\beta=90°\Rightarrow$$

$$\Rightarrow(x+y)\perp(x-y).$$

Proved.

<u>Answer:</u> Proved.