Conditions

show that the area of the triangle formed by the points (x1,y1,z1),(x2,y2,z2) and the origin is 1/2sqrt((y1z2-y2z1)2+(z1x2-z2x1)2+(x1y2-x2y1)2)

Solution

We must find the area of the triangle, which has 3 top points:

$A=(x_1,y_1,z_1), B=(x_2,y_2,z_2), C=(0,0,0)$

As it known, there is a formula to find the area of triangle using coordinates of the tops:

$$S=\sqrt{S_x^2+S_y^2+S_z^2}$$

$$S_x=\frac{1}{2}\begin{vmatrix} y_B-y_A & z_B-z_A \\ y_C-y_A & z_C-z_A \end{vmatrix}=\frac{1}{2}\begin{vmatrix} 1 & y_A & z_A \\ 1 & y_B & z_B \\ 1 & y_C & z_C \end{vmatrix}$$

$$S_y=\frac{1}{2}\begin{vmatrix} x_A & 1 & z_A \\ x_B & 1 & z_B \\ x_C & 1 & z_C \end{vmatrix}, \qquad S_z=\frac{1}{2}\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix}$$

Where $\mathbf{r}_B(x_B,y_B,z_B)$, $\mathbf{r}_C(x_C,y_C,z_C)$, $\mathbf{r}_A(x_A,y_A,z_A)$ - top points.

So,

$$S_A=\frac{1}{2}\begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & 0 & 0 \end{vmatrix}=\frac{1}{2}(0+y_1z_2+0-y_2z_1-0-0)=\frac{1}{2}(y_1z_2-y_2z_1)$$

$$S_B=\frac{1}{2}\begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ 0 & 1 & 0 \end{vmatrix}=\frac{1}{2}(0+z_1x_2+0-z_2x_1-0-0)=\frac{1}{2}(z_1x_2-z_2x_1)$$

$$S_B=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ 0 & 0 & 1 \end{vmatrix}=\frac{1}{2}(0+x_1y_2+0-x_2y_1-0-0)=\frac{1}{2}(x_1y_2-x_2y_1)$$

$$S=\sqrt{\frac{1}{4}(z_1x_2-z_2x_1)^2+\frac{1}{4}(z_1x_2-z_2x_1)^2+\frac{1}{4}(x_1y_2-x_2y_1)^2}$$
$$=\frac{1}{2}\sqrt{(z_1x_2-z_2x_1)^2+(z_1x_2-z_2x_1)^2+(x_1y_2-x_2y_1)^2}$$

Q.E.D.