

Question 21163

$$\frac{\tan(45^\circ - \tan(30^\circ))}{1 - \tan(45^\circ - \tan(30^\circ))} =$$

**Solution.**

We first evaluate the expression  $\tan(45^\circ - \tan(30^\circ))$ .

$$\tan(45^\circ - \tan(30^\circ)) = \frac{\tan(45^\circ) - \tan(\tan(30^\circ))}{1 + \tan(45^\circ)\tan(\tan(30^\circ))} = \frac{1 - \tan(\tan(30^\circ))}{1 + \tan(\tan(30^\circ))}. \text{ Here we use that } \tan(45^\circ) = 1.$$

Thus,

$$1 - \tan(45^\circ - \tan(30^\circ)) = 1 - \frac{1 - \tan(\tan(30^\circ))}{1 + \tan(\tan(30^\circ))} = \frac{2\tan(\tan(30^\circ))}{1 + \tan(\tan(30^\circ))}.$$

Finally,

$$\frac{\tan(45^\circ - \tan(30^\circ))}{1 - \tan(45^\circ - \tan(30^\circ))} = \frac{1 - \tan(\tan(30^\circ))}{1 + \tan(\tan(30^\circ))} \cdot \frac{1 + \tan(\tan(30^\circ))}{2\tan(\tan(30^\circ))} = \frac{1 - \tan(\tan(30^\circ))}{2\tan(\tan(30^\circ))} = \frac{1 - \tan(\frac{\sqrt{3}}{3})}{2\tan(\frac{\sqrt{3}}{3})}.$$

**Answer.**  $\frac{1 - \tan(\frac{\sqrt{3}}{3})}{2\tan(\frac{\sqrt{3}}{3})}$ .