

$$f(x) = x, \quad x \in [0, 2\pi]$$

The fourier series is:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} x dx = \frac{1}{\pi} \frac{x^2}{2} \Big|_0^{2\pi} = \frac{1}{2\pi} 4\pi^2 = 2\pi$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} x \cos(nx) = \frac{1}{\pi} \frac{\cos(2\pi n) - 1 + 2\pi n \sin(2\pi n)}{n^2} = 0$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} x \sin(nx) = -\frac{1}{\pi} \frac{2\pi n \cos(2\pi n) + \sin(2\pi n)}{n^2} = -\frac{2}{n}$$

$$f(x) = \pi - \sum_{n=1}^{+\infty} \frac{2}{n} \sin nx$$