

Question 1. Let G be a finite group. Let p and q be prime numbers. Assume that there exist elements in G , say a and b , of orders p and q , respectively. Prove that order of G is a multiple of pq .

Solution. It follows from Lagrange theorem, that the order of an element of a finite group divides the order of this group. Let the order of G be $n \in \mathbb{N}$. Since G contains a of order p and b of order q , then p and q divide n . Therefore, n is a multiple of the least common factor of p and q . But p and q are relatively prime, so their least common factor is their product pq . Thus, n is a multiple of pq . \square