Question 1. *Let G be a group, let H be a normal subgroup of G. Prove that if a is an element of G, then the order of Ha in G/H is a divisor of order of a in G.*

Solution. Suppose the order of a in G is finite and equals $n \in \mathbb{N}$. So, $a^n = 1$ in G. Then in the factor group G/H we have

$$(Ha)^n = Ha^n = H \cdot 1 = H,$$

which is the identity of G/H. Therefore, the order of Ha in G/H, which is the smallest $k \in \mathbb{N}$ such that $(Ha)^k = H$, divides n, i. e. the order of a.

If the order of a is infinite, the order of Ha however can be finite. For example, take $G = \mathbb{Z}$, $H = 2\mathbb{Z}$ and $a = 2$. Then $Ha = H$, so Ha has the order 1 in G/H, but $na \neq 0$ for all $n \in \mathbb{N}$, $n \neq 0$. $\square$