

How can we find the value of  $\sin 3$ ,  $\sin 6$ ,  $\sin 9$ .

**Solution:**

Using that

$$\sin 18 = \frac{\sqrt{5}-1}{4} \qquad \cos 18 = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\sin 15 = \frac{\sqrt{6}-\sqrt{2}}{4} \qquad \cos 15 = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin 3 = \sin(18 - 15) = \sin 18 * \cos 15 - \cos 18 * \sin 15$$

$$\sin 3 = \left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) - \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$$

**This is exact value of  $\sin 3$**

If we need find  $\sin 6$ , by the double-angle formula for sine we have

$$\sin 6 = 2 \sin 3 * \cos 3$$

So we need find value of  $\cos 3$

$$\cos 3 = \cos(18 - 15) = \cos 18 * \cos 15 + \sin 18 * \sin 15$$

$$\cos 3 = \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + \left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$$

$$\sin 6 = 2 \sin 3 * \cos 3$$

$$\begin{aligned} &= 2 \left[ \left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) - \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) \right] \\ &* \left[ \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + \left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) \right] \end{aligned}$$

**This is exact value of  $\sin 6$**

If we need find  $\sin 9$  we will use formula  $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

$$\sin 9 = 3\sin 3 - 4\sin^3 3$$

$$\sin 9 = 3 \left[ \left( \frac{\sqrt{5}-1}{4} \right) \left( \frac{\sqrt{6}+\sqrt{2}}{4} \right) - \left( \frac{\sqrt{10+2\sqrt{5}}}{4} \right) \left( \frac{\sqrt{6}-\sqrt{2}}{4} \right) \right] \\ - 4 \left[ \left( \frac{\sqrt{5}-1}{4} \right) \left( \frac{\sqrt{6}+\sqrt{2}}{4} \right) - \left( \frac{\sqrt{10+2\sqrt{5}}}{4} \right) \left( \frac{\sqrt{6}-\sqrt{2}}{4} \right) \right]^3$$

**This is exact value of  $\sin 9$**

**NOTE:**

If you don't know how to find  $\sin 15$ ,  $\cos 15$  for example you can do such:

$$\sin 15 = \sin(45 - 30) = \sin 45 * \cos 30 - \cos 45 * \sin 30$$

$$\cos 15 = \cos(45 - 30) = \cos 45 * \cos 30 + \sin 45 * \sin 30$$