Conditions

what is the integral of 1/(sqrt(-x^(2)-6x-5)) and arctan sqrt(x)? thank you!

Solution

1.

$$\int \frac{dx}{\sqrt{-x^2-6x-5}} = \int \frac{dx}{4-(x+3)^2} = \begin{bmatrix} u = x+3 \\ du = dx \end{bmatrix} = \int \frac{du}{4-u^2} = \int \frac{du}{2\sqrt{1-\frac{u^2}{4}}} = \begin{bmatrix} s = \frac{u}{2} \\ du = ds \end{bmatrix} =$$

$$= \int \frac{ds}{\sqrt{1-s^2}} = sin^{-1}s + c = sin^{-1}\left(\frac{x+3}{2}\right) + c = \boldsymbol{arcsin\left(\frac{x+3}{2}\right) + c}$$

2.

$$\int \arctan\sqrt{x}\,dx = \begin{bmatrix} u = \sqrt{x} \\ du = \frac{dx}{2\sqrt{x}} \end{bmatrix} = \int u \arctan u\,du = \begin{bmatrix} f = \arctan u & df = \frac{du}{u^2+1} \\ g = \frac{u^2}{2} & dg = udu \end{bmatrix} =$$

$$= u^2 \arctan u - 2\int \frac{u^2}{2(u^2+1)}du = u^2 \arctan u - \int \left(1 - \frac{1}{(u^2+1)}\right) du$$

$$= u^2 \arctan u + \arctan u - u + c = x \arctan\sqrt{x} + \arctan\sqrt{x} - \sqrt{x} + c$$

$$= \boldsymbol{(x+1)\arctan\sqrt{x} - \sqrt{x} + c}$$