Conditions

If f is continuous on [a, b] and integral a to b of f(x)g(x)dx=0 for all continuous functions g on [a, b] then f is identically equal to 0 on [a, b]

Solution

This is not true. Consider the counterexample:

$f(x) = sinx$

$g(x) = 1$

$x\epsilon[a, b] = [0, 2\pi]$

Then f, g are continuous on [a, b] and

$$\int_0^{2\pi} \sin x \, dx = 0$$

but
$f(x) = sinx \neq 0 \; \forall x \in [0, 2\pi]$