Prove the following: if r and s are rational numbers then, $r - s$ is rational

Proof.

Suppose r and s are rational numbers. [We must show that $r - s$ is rational.] Then, by the definition of rational numbers, we have

$$r = \frac{a}{b} \text{ for some integers a and b with } b \neq 0.$$

$$s = \frac{c}{d} \text{ for some integers c and d with } d \neq 0.$$

$$r - s = \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

Now, let $p = ad - bc$ and $q = bd$. Then, p and q are integers [because products and sums(differences) of integers are integers and because a, b, c and d are all integers. Also, $q \neq 0$ by zero product property] Hence,

$$r - s = \frac{p}{q}, \text{ where p and q are integers and } q \neq 0.$$

Therefore, by definition of a rational number, $r - s$ is rational.

This is what was to be shown.