1. Find the inverse of the following function. Find the domain, range, and asymptotes of each function. Graph both functions on the same coordinate plane.

$$F(x) = 5 + e^{(-\frac{x}{2})}$$

Solution:

The function $f(x) = 5 + e^{(-\frac{x}{2})}$is a one-to-one function, so it has an inverse on its domain. The domain is all real numbers. The range is (5; ∞).

Set of values of function$e^{(-\frac{x}{2})}$, as well as any exponential function is the interval (0, +∞). Therefore, the graph of is above the axis Ox, (The range of is (0, +∞). It is an exponential decrease function.)

We need to interchange the x and the y and then solve for y. This means that we are solving for y in. Find the inverse is to exchange the x and y, and then solve for y:

$y = 5 + e^{(-\frac{x}{2})}$ - is the original function;

$x = 5 + e^{(-\frac{y}{2})}$ - will give the inverse relation. Solving for y we get:

$e^{(-\frac{y}{2})} = x - 5$

$\ln\left(e^{(-\frac{y}{2})}\right) = \ln(x - 5)$

$-\frac{y}{2} = \ln(x - 5)$

$-\mathrm{y} = 2 \cdot \ln(x - 5)$

$\mathrm{y} = -2 \cdot \ln(x - 5)$

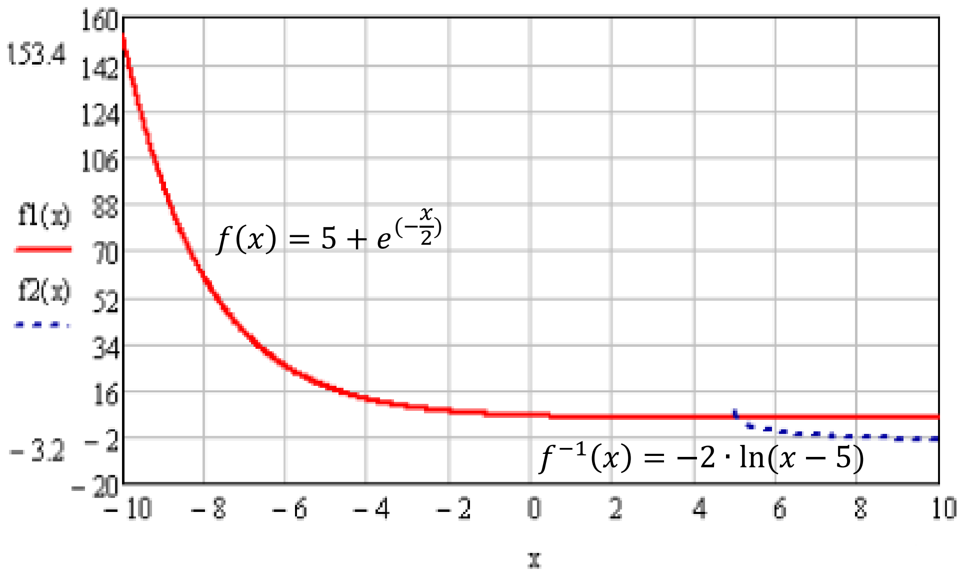
The domain for this function is (5; ∞)as expected - a function's inverse has for its domain the range of the function, and the range of the inverse is the domain of the original function. Thus the range of the inverse is all real numbers.

The domain of is restricted only by the logarithm function, whose argument must be greater than zero; thus x must be greater than zero for $\mathrm{x} - 5 > 0 => x > 5, x \in (5; \infty)$. The range of the logarithm function is all real numbers.

If $f(x) = 5 + e^{(-\frac{x}{2})}$ then$f^{-1}(x) = -2 \cdot \ln(x - 5)$.

The domain of $f(x)$ is all real numbers - the domain of the inverse is$(5; \infty)$. The range of $f(x)$ is$(5; \infty)$, while the range of the inverse is all real numbers.

The graph of $f(x) = 5 + e^{(-\frac{x}{2})}$ in red. Inverse function in blue. Note that $f(x)$ has a horizontal asymptote of y=5 as x grows without bound, and that the inverse has a vertical asymptote at x=5:



160
153.4
142
124
106
f1(x)
88
70
f2(x)
52
34
16
− 3.2
− 2
− 20
− 10
− 8
− 6
− 4
− 2
0
2
4
6
8
10
x
$f(x) = 5 + e^{(-\frac{x}{2})}$
$f^{-1}(x) = -2 \cdot \ln(x - 5)$