1. What is the integral of a) $\frac{1}{\sqrt{-x^2-6x-5}}$ and b) $\arctan\sqrt{x}$?

a) Intermediate steps:

$$\int \frac{1}{\sqrt{-x^2-6x-5}}dx$$

For the integrand $\frac{1}{\sqrt{-x^2-6x-5}}$, complete the square

$$= \int \frac{1}{\sqrt{4-(x+3)^2}}dx$$

For the integrand $\frac{1}{\sqrt{4-(x+3)^2}}$, substitute $u = x + 3$ and $du = dx$:

$$= \int \frac{1}{\sqrt{4-u^2}}du$$

Factor 4 out of the radical:

$$= \int \frac{1}{2\sqrt{1-\frac{u^2}{4}}}du$$

Factor out constants:

$$= \frac{1}{2}\int \frac{1}{\sqrt{1-\frac{u^2}{4}}}du$$

For the integrand $\frac{1}{\sqrt{1-\frac{u^2}{4}}}$, substitute $s = \frac{u}{2}$ and $ds = \frac{1}{2}du$:

$$= \int \frac{1}{\sqrt{1-s^2}}ds$$

The integral of $\frac{1}{\sqrt{1-s^2}}$ is $\arcsin(\mathrm{s})$:

$$= \arcsin(\mathrm{s}) + C$$

Substitute back for $s = \frac{u}{2}$:

$$= \arcsin(\frac{u}{2}) + C$$

Substitute back for $u = x + 3$:

Answer:

$$\int \frac{1}{\sqrt{-x^2-6x-5}}dx = \arcsin(\frac{x+3}{2}) + C$$

b) Intermediate steps:

$$\int \arctan(\sqrt{x})\,dx$$

For the integrand $\arctan(\sqrt{x})$, substitute $u = \sqrt{x}$ and $du = \frac{1}{2\sqrt{x}}dx$:

$$= 2 \cdot \int u \cdot \arctan(u)\,du$$

For the integrand $u \cdot \arctan(u)$

, integrate by parts, $\int f\,dg = f \cdot g - \int g\,df$, where

$f = \arctan(u), \qquad dg = u\,du,$

$df = \frac{1}{u^2+1}du, \qquad g = \frac{u^2}{2}$:

$$= u^2 \cdot \arctan(u) - 2 \cdot \int \frac{u^2}{2 \cdot (u^2+1)}du$$

Factor out constants:

$$= u^2 \cdot \arctan(u) - \int \frac{u^2}{u^2+1}du$$

For the integrand $\frac{u^2}{u^2+1}$, do long division:

$$= u^2 \cdot \arctan(u) - \int (1 - \frac{u^2}{u^2+1})\,du$$

Integrate the sum term by term and factor out constants:

$$= u^2 \cdot \arctan(u) + \int \frac{1}{u^2+1}du - \int 1\,du$$

The integral of $\frac{1}{u^2+1}$ is $\arctan(u)$:

The integral of 1 is u:

$$= u^2 \cdot \arctan(u) + \arctan(u) - u + C$$

Substitute back for $u = \sqrt{x}$:

$$= x \cdot \arctan(\sqrt{x}) + \arctan(\sqrt{x}) - \sqrt{x} + C$$

Factor the answer a different way:

Answer:

$$= (x + 1) \cdot \arctan(\sqrt{x}) - \sqrt{x} + C$$