Conditions

For each of the following relations, indicate whether they are reflexive, symmetric, and transitive (for example select reflexive and transitive if the relation is reflexive and transitive, but not symmetric).

(a) Let R be the relation on N given by xRy if and only if x divides y.
(b) Let X be a set and let R be the relation "⊆" on . In other words, SRT if and only if S⊆T.

Solution

The relation R is called reflexive on a set X, if:

$$\forall a \epsilon X \; aRa$$

The relation R is called symmetric on a set X, if:

$$\forall a, b \epsilon X \; aRb \; \rightarrow bRa$$

The relation R is called transitive on a set X, if:

$$\forall a, b, c \epsilon X \; aRb \text{ and } bRc \; \rightarrow aRc$$

a)

First of all, we must notice, that every number of N can divide every number of N. But we will check these properties, meaning **integer division**.

$$R: \forall x, y \epsilon N: xRy \; \leftrightarrow \frac{y}{x} \epsilon N$$

The relation R is reflexive:

$$\forall x \epsilon N: xRx \; \leftrightarrow \frac{x}{x} = 1 \epsilon N$$

The relation R is not symmetric:

$$\exists \, x = 5, y = 10, x, y \epsilon N : xRy = \frac{10}{5} = 2 \epsilon N, but \; yRx = \frac{5}{10} = 0.5 \epsilon Q \backslash N$$

The relation R is transitive:

$$\forall a, b, c \epsilon N \; aRb \text{ and } bRc \; \rightarrow b = ta, t \epsilon N, c = kb, k \epsilon N$$

Consider

$$aRc = \frac{c}{a} = \frac{kb}{\frac{b}{t}} = kt.$$

As $k, t \in N, \rightarrow kt \in N$. And aRc

b)

$R: \forall S, T \in X\ SRT \leftrightarrow S \subseteq \mathrm{T}$

The relation R is reflexive:

$\forall S \in X\ SRS = S \subseteq \mathrm{S}$

The relation R is not symmetric:

$\exists\ S, T \in X, M \in X, M \cap S = \emptyset : T = S \cup M$. Then $S \subseteq \mathrm{T}$, but T is not $\subseteq \mathrm{S}$, as it consist M

The relation R is transitive:

$\forall A, B, C \in X\ ARB$ and $BRC \rightarrow A \subseteq \mathrm{B}, \mathrm{B} \subseteq \mathrm{C}$

Each element from A is in B, and each element from B is in C. Then each element of A is in C.