Equation $\frac{dy}{dx}=f(x,y)$ is homogeneous if the function f(x,y) is homogeneous, that is

$f(tx,ty)=f(x,y)$ for any number t.

$$(x^2+xy)dy=(x^2+y^2)dx, \qquad \frac{dy}{dx}=\frac{x^2+y^2}{x^2+xy}$$

$f(tx,ty)=\frac{t^2x^2+t^2y^2}{t^2x^2+t^2xy}=\frac{x^2+y^2}{x^2+xy}=f(x,y)$ - equation is homogeneous.

Let $y=ux$, then $\frac{dy}{dx}=\frac{du}{dx}x+u$ and $\frac{dy}{dx}=\frac{x^2+y^2}{x^2+xy}=\frac{x^2+u^2x^2}{x^2+ux}=\frac{1+u^2}{1+u}$

$\frac{du}{dx}x+u=\frac{1+u^2}{1+u}$, $\frac{du}{dx}x=\frac{1+u^2}{1+u}-u=\frac{1-u}{1+u}$, $\frac{dx}{x}=\frac{1+u}{1-u}du$,

$\frac{dx}{x}=\frac{1-u+2u}{1-u}du$, $\frac{dx}{x}=(1-2(1+\frac{1}{u-1}))du$

$\int\frac{dx}{x}=\int(1-2(1+\frac{1}{u-1}))du$, $\ln(x)+c=-u-2\ln(u-1)$,

$u=-\ln(Ax)-2\ln(u-1)=-\ln(Ax(u-1)^2)$, $A=e^c=const$

$$\frac{y}{x}=-\ln\left(Ax\left(\frac{y}{x}-1\right)^2\right)$$