Task:

Solve

$$2\cos^2\left(\frac{x}{3}\right) + 3\sin\left(\frac{x}{3}\right) - 3 = 0$$

Solution:

$$2\cos^2\left(\frac{x}{3}\right) + 3\sin\left(\frac{x}{3}\right) - 3 = 0$$

$$2\left(1 - \sin^2\left(\frac{x}{3}\right)\right) + 3\sin\left(\frac{x}{3}\right) - 3 = 0$$

$$-2\sin^2\left(\frac{x}{3}\right) + 3\sin\left(\frac{x}{3}\right) - 1 = 0$$

$$2\sin^2\left(\frac{x}{3}\right) - 3\sin\left(\frac{x}{3}\right) + 1 = 0$$

$$D = 9 - 2*4 = 1$$

$$\sin\left(\frac{x}{3}\right) = \frac{3 \pm 1}{4}$$

$$\left[\begin{array}{l} \sin\left(\frac{x}{3}\right) = 1 \\ \sin\left(\frac{x}{3}\right) = \frac{1}{2} \end{array}\right. \Longrightarrow \left[\begin{array}{l} \frac{x}{3} = \frac{\pi}{2} + 2\pi n, \ n \in Z \\ \frac{x}{3} = \frac{\pi}{6} + 2\pi k, \ k \in Z \\ \frac{x}{3} = \frac{5\pi}{6} + 2\pi m, \ m \in Z \end{array}\right. \Longrightarrow \left[\begin{array}{l} x = \frac{3\pi}{2} + 6\pi n, \ n \in Z \\ x = \frac{\pi}{2} + 6\pi k, \ k \in Z \\ x = \frac{5\pi}{2} + 6\pi m, \ m \in Z \end{array}\right.$$

Answer: $\frac{\pi}{2} + 6\pi k,$ $\frac{3\pi}{2} + 6\pi n,$ $\frac{5\pi}{2} + 6\pi m \ ;$ $n, m, k \in Z$