## Conditions

Let A be a commutative ring with unity and let F be a field. Let f be a homomorphism from A onto F. Prove that ker f is a maximal ideal in A.

## Solution

As A is a commutative ring with unity, then:

$\exists e \epsilon A : \forall a \epsilon A \ a \cdot e = e \cdot a = a$

And

$\forall a, b \ \epsilon \ A \ a \cdot b = b \cdot a$

As F is a field, then it's a commutative ring with unity which is not equal to 0.

$$f : A \to F : \forall a, b \ \epsilon \ A \ f(a \cdot b) = f(a) \cdot f(b)$$

$\ker(f) = \{x \epsilon A : f(x) = 0\}$

We know a theorem: M is a maximal ideal ⇐⇒ A/M is a field.

And as the F is field, then A/$\ker(f)$ is a field.

That's why ker(f) is a maximal ideal