

Conditions

Consider the probability experiment of a sequence of independent trials (number of trials $n=11$), each trial is the rolling of a pair of 6-sided dice. suppose a random event A is the sum of the roll is an odd number and a random variable represents the number of successes that occur in the n trials. Find the probability of the success for a single trial, find the probability mass function $P_n(m)$, $m=1,n$, find the mathematical expectation and variance of the random variable

Solution

Let's random variable ξ is represents the number of successes that occur in the n trials.

Look at the table below:

ξ	0	1	2	3	4	5	6	7	8	9	10	11
p	?	?	?	?	?	?	?	?	?	?	?	?

Using Bernoulli's formula to find p for each number of success.

$$P_{n,m} = C_n^m p^m q^{n-m}$$

$$P_{11,0} = C_{12}^0 p^0 q^{11} = \frac{11!}{10!0!} \left(\frac{1}{2}\right)^{11} = 0.00049$$

$$P_{11,1} = C_{12}^1 p^1 q^{10} = 0.00537$$

...

$$P_{11,11} = 0.00049$$

ξ	0	1	2	3	4	5	6	7	8	9	10	11
p	0.00049	0.00537	0.02686	0.08057	0.16113	0.22559	0.22559	0.16113	0.08057	0.02686	0.00537	0.00049

$$M(\xi) = \sum_{i=0}^{11} \xi_i x_i = 5.5$$

$$D(\xi) = M(\xi^2) - (M(\xi))^2$$

ξ^2	0	1	4	9	16	25	36	49	64	81	100	121
p	0.00049	0.00537	0.02686	0.08057	0.16113	0.22559	0.22559	0.16113	0.08057	0.02686	0.00537	0.00049

$$D(\xi) = 33 - 5.5^2 = 2,75$$