Solve each absolute value equation.

1. $|7x + 2| = 10$

2. $|3x - 5| = -1$

3. $|x^2 - 2x - -16| = 8$

4. $|3x - 2| = 7$

5. $|x^2 + 2x + 1| = -4$

6. $|3x + 2| + 3 = 0$

7. $|3x - 1| = |x + 4|$

8. $|\frac{3}{k-1}| = 4$

9. $|x^2 - 3x + 3| = 3$

Solution:

1. $|7x + 2| = 10$

$$\begin{bmatrix} 7x + 2 = 10 \\ 7x + 2 = -10 \end{bmatrix} \Rightarrow \begin{bmatrix} x = \frac{8}{7} \\ x = -\frac{12}{7} \end{bmatrix} \Rightarrow \begin{bmatrix} x = 1\frac{1}{7} \\ x = -1\frac{5}{7} \end{bmatrix}$$

Answer: $1\frac{1}{7}$, $-1\frac{5}{7}$

2. $|3x - 5| = -1$

It has no solution, because absolute value must be only positive.

Answer: no solution

3. $|x^2 - 2x - 16| = 8$

$$\begin{bmatrix} x^2 - 2x - 16 = 8 \\ x^2 - 2x - 16 = -8 \end{bmatrix} \Rightarrow \begin{bmatrix} x^2 - 2x - 24 = 0 \\ x^2 - 2x - 8 = 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 = 6, & x_2 = -4 \\ x_3 = 4, & x_2 = -2 \end{bmatrix}$$

Answer: $-4;\ -2;\ \ 4;\ \ 6.$

4. $|3x - 2| = 7$

$$\begin{bmatrix} 3x - 2 = 7 \\ 3x - 2 = -7 \end{bmatrix} \Rightarrow \begin{bmatrix} x = 3 \\ x = -\frac{5}{3} \end{bmatrix} \Rightarrow \begin{bmatrix} x = 3 \\ x = -1\frac{2}{3} \end{bmatrix}$$

Answer: $3;\ -1\frac{2}{3}$.

5. $|x^2 + 2x + 1| = -4$

It has no solution, because absolute value must be only positive.

Answer: no solution

6. $|3x + 2| + 3 = 0$

$|3x + 2| = -3$

It has no solution, because absolute value must be only positive.

Answer: no solution

7. $|3x - 1| = |x + 4|$

$$\begin{bmatrix} 3x - 1 = x + 4 \\ 3x - 1 = -x - 4 \end{bmatrix} \Rightarrow \begin{bmatrix} x = \frac{5}{2} \\ x = -\frac{3}{4} \end{bmatrix} \Rightarrow \begin{bmatrix} x = 2\frac{1}{2} \\ x = -\frac{3}{4} \end{bmatrix}$$

Answer: $-\frac{3}{4};\ 2\frac{1}{2}$

8. $\left|\frac{3}{k - 1}\right| = 4$

$$\begin{bmatrix} \frac{3}{k-1} = 4 \\ \frac{3}{k-1} = -4 \end{bmatrix} \Rightarrow \begin{bmatrix} k = \frac{3}{4} + 1 \\ k = -\frac{3}{4} + 1 \end{bmatrix} \Rightarrow \begin{bmatrix} k = 1\frac{3}{4} \\ k = \frac{1}{4} \end{bmatrix}$$

Answer: $\frac{1}{4};\ 1\frac{3}{4}$

9. $|x^2 - 3x + 3| = 3$

$$\begin{bmatrix} x^2 - 3x + 3 = 3 \\ x^2 - 3x + 3 = -3 \end{bmatrix} \Rightarrow \begin{bmatrix} x^2 - 3x = 0 \\ x^2 - 3x + 6 = 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 = 0, \quad x_2 = 3 \\ D < 0 \Rightarrow no\ solution \end{bmatrix}$$

Answer: 0; **3**