Conditions

Compute lower and upper integrals o0f the function
f(x)
=4, 1<=x<=2
=3, 2<x<=4
=2, 4<x<=5

Is this function Riemann integrable on [1,5]? Justify.

Solution

$$f = \begin{cases} 4, 1 \le x \le 2 \\ 3, 2 < x \le 4 \\ 2, 4 < x \le 5 \end{cases}$$

Here we must use the Darboux integral theory.

The lower Darboux sum:

$$s(f,\tau) = \sum_{k=1}^{n} m_k (x_k - x_{k-1})$$

The upper Darboux sum:

$$S(f,\tau) = \sum_{k=1}^{n} M_k (x_k - x_{k-1})$$

Where:

$$m_k = \inf \{f(x) : x \in [x_{k-1}, x_k]\}, k = \overline{1,n}$$
$$M_k = \sup \{f(x) : x \in [x_{k-1}, x_k]\}, k = \overline{1,n}$$
$$\tau = \{x_k\}_{k=0}^{n} : a = x_0 < x_1 < \ldots < x_{n-1} < x_n = b$$

For our case:

$$\tau : a = 1 < 2 < 4 < 5 = b$$

$$s(f,\tau) = 4 \cdot (2-1) + 3(4-2) + 2(5-4) = 4 + 6 + 2 = 12$$

$$S(f,\tau) = 4 \cdot (2-1) + 3(4-2) + 2(5-4) = 12$$

$$\forall \tau \ \ s(f,\tau) \le I_*(f) \le I^*(f) \le S(f,\tau)$$

As we have equal $s(f,\tau)$and $S(f,\tau)$, then upper and lower integrals are equal to 12.

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$$\int_1^5 f(x)dx = \int_1^2 4dx + \int_2^4 3dx + \int_4^5 2dx = 8 - 4 + 12 - 6 + 10 - 8 = 12$$

As

$$I^*(f) = I_*(f) = \int_a^b f(x)\,dx$$

Then function is integrable on [1,5] by the Darboux criterion.