Conditions

Explain how to factor the following trinomials forms: $x^2 + bx + c$ and $ax^2 + bx + c$. Is there more than one way to factor this?

Solution

A quadratic equation with real or complex coefficients has two solutions, called roots. These two solutions may or may not be distinct, and they may or may not be real.

Having

$$ax^2 + bx + c = 0$$

the roots are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where the symbol "±" indicates that both

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

are solutions of the quadratic equation.

The factorization of trinomial form $ax^2 + bx + c$ is below:

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

Where x_1, x_2 are the roots of equation

$$ax^2 + bx + c = 0$$

For a=1 we have a form for trinomial $x^2 + bx + c = (x - x_1)(x - x_2)$

There is one more method how to find the roots of trinomial a$x^2 + bx + c$

This method is called Vieta's formula.

According to this formula, if x_1, x_2 are the roots of equation

$$ax^2 + bx + c = 0$$

Then they can be found from a system:

$$\begin{cases} x_1 + x_2 = -\dfrac{b}{a} \\ x_1 x_2 = \dfrac{c}{a} \end{cases}$$