Conditions

The probability that a certain type of missile hits a target is 0.4 . Five missiles are fired at this target. Determine:
a) the probability that the target is hit exactly twice.
b) the probability of missing the target on all five attempts.
c) the probability of hitting the target at least once.
d) the expected number of times that the target will be hit and calculate the standard deviation

Solution

a)

This is a probability when 2 times missile hits a target and 3 times misses:

$$P = C_5^2 \cdot 0.4 \cdot 0.4 \cdot 0.6 \cdot 0.6 \cdot 0.6 = \frac{5!}{2!\,3!} 0{,}03456 = 0{,}3456$$

b)

This is a probability when missile misses 5 times in a row:

$$P = 0.6 \cdot 0.6 \cdot 0.6 \cdot 0.6 \cdot 0.6 = 0{,}07776$$

c)

This is a probability when missile must hit a target once or more times:

$$P = 1 - Q$$

where Q is a probability that there were no hits at all

$$Q = 0{,}07776$$

$$P = 1 - 0{,}07776 = 0{,}92224$$

d) Let's build a probability distribution for a random value ξ, which is the amount of hitting targets (by using a Bernoulli formula) :

x	0	1	2	3	4	5
P	0,07776	0,2592	0,3456	0,2304	0,0768	0,01024

The expected number of times that the target will be hit is a mean of this distribution:

$$\boldsymbol{M(\xi)} = \sum_{i=1}^{5} x_i p_i = 0 \cdot 0{,}07776 + 1 \cdot 0{,}2592 + 2 \cdot 0{,}3456 + 3 \cdot 0{,}2304 + 4 \cdot 0{,}0768 + 5 \cdot 0{,}01024 = 0{,}2592 + 0{,}6912 + 0{,}6912 + 0{,}3072 + 0{,}0512 = \mathbf{1{,}37792}$$

The standard deviation is:

$$\sigma(\xi) = \sqrt{M(\xi^2) - \mathrm{M}^2(\xi)}$$

The distribution for ξ^2:

x	0	1	4	9	16	25
P	0,07776	0,2592	0,3456	0,2304	0,0768	0,01024

$$M(\xi^2) = 0{,}2592 + 1{,}3824 + 2{,}0736 + 1{,}2288 + 0{,}256 = 5{,}2$$

$$\boldsymbol{\sigma(\xi)} = \sqrt{M(\xi^2) - \mathrm{M}^2(\xi)} = \sqrt{5.2 - 1{,}899} \approx \mathbf{1{,}817}$$