

A researcher is interested in comparing the Christian maturity level of students who volunteer for community service versus those who do not. The researcher assumes that those who perform community service will have higher Christian maturity scores. The maturity scores tend to be skewed (not normally distributed). Higher scores indicate higher Christian maturity.

No Community Service Community Service

32 47
 40 48
 54 59
 13 72
 20 80
 26 55

- a. What statistical test should be used to analyze these data?
- b. Is this a one- or two- tailed test?
- c. Identify H0 and Ha for this study
- d. Conduct the appropriate analysis
- e. Should H0 be rejected? What should the researcher conclude?

Solution:

The student's t test for equality means can be used.

Let μ_1 and μ_2 denote the mean scores of the no community service and community service groups respectively.

H0: $\mu_1 = \mu_2$ (the mean scores are the same in both groups)

H1: $\mu_1 < \mu_2$ (the mean score of the community service is greater than that of the non community service group)

x1	x2	x1 ²	x2 ²
32	47	1024	2209
40	48	1600	2304
54	59	2916	3481
13	72	169	5184
20	80	400	6400
26	55	676	3025
185	361	6785	22603

$$n_1 = n_2 = 6, \sum x_1 = 185, \sum x_2 = 361, \sum x_1^2 = 6785, \sum x_2^2 = 22603$$

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{185}{6} = 30.83, \bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{361}{6} = 60.17$$

$$s_1^2 = \frac{\sum x_1^2 - (\sum x_1)^2/n_1}{n_1 - 1} = \frac{6785 - 185^2/6}{5} = 216.17$$

$$s_2^2 = \frac{\sum x_2^2 - (\sum x_2)^2/n_2}{n_2 - 1} = \frac{22603 - 361^2/6}{5} = 176.57$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{30.83 - 60.17}{\sqrt{\frac{5 \times 216.17 + 5 \times 176.57}{6 + 6 - 2} \left(\frac{1}{6} + \frac{1}{6}\right)}} = -3.6257$$

Degrees of freedom = 6+6-2=10

The p-value is 0.0023 (left tail test)

Since the p-value is less than 0.01, the t is significant at the 1% level.

So we reject the null hypothesis.

Conclusion: those who perform community service will have higher Christian maturity scores.

When the assumption of normality is not satisfied, a non-parametric alternative is Wilcoxon Rank-Sum Test.

Wilcoxon Rank-Sum test

data	rank	data	rank
32	4	47	6
40	5	48	7
54	8	59	10
13	1	72	11
20	2	80	12
26	3	55	9
	23		55

The pooled data is ranked.

$$W_1 = \text{sum of the ranks for the first group} = 4 + 5 + 8 + 1 + 2 + 3 = 23$$

$$W_2 = \text{sum of the ranks for the second group} = 6 + 7 + 10 + 11 + 12 + 9 = 55$$

$$U_1 = W_1 - \frac{n_1(n_1 + 1)}{2} = 23 - \frac{6 \times 7}{2} = 23 - 21 = 2$$

The critical region is $U_1 \leq 4$

The computed value falls in the critical region. So we reject the null hypothesis.

Answer: Those who perform community service will have higher Christian maturity scores.