

1. Antiderivate of $\ln(x^2 + 1)dx$

Solution:

$$\begin{aligned}\int \ln(x^2 + 1)dx &= \left| \begin{array}{l} u = \ln(x^2 + 1), du = \frac{2xdx}{x^2 + 1} \\ dv = dx, v = x \end{array} \right| \\ &= x \ln(x^2 + 1) \\ &\quad - 2 \int \frac{x^2 dx}{x^2 + 1} = x \ln(x^2 + 1) - 2 \left(\int 1 - \frac{1}{x^2 + 1} \right) dx \\ &= x \ln(x^2 + 1) - 2(x - \text{atan}(x)) + C,\end{aligned}$$

where $C = \text{const}$

Answer: $x \ln(x^2 + 1) - 2(x - \text{atan}(x)) + C.$

2. Antiderivate of $\frac{e^{2x}}{\sqrt{1-e^x}} dx$

Solution:

$$\begin{aligned}\int \frac{e^{2x}}{\sqrt{1-e^x}} dx &= \int \frac{e^x}{\sqrt{1-e^x}} d(e^x) = |t = e^x| = \int \frac{tdt}{\sqrt{1-t}} \\ &= \left| \begin{array}{l} u = t, du = dt \\ dv = \frac{dt}{\sqrt{1-t}}, v = -2\sqrt{1-t} \end{array} \right| \\ &= -2t\sqrt{1-t} + 2 \int \sqrt{1-t} dt = -2t\sqrt{1-t} - \frac{4}{3} \sqrt{(1-t)^3} + C\end{aligned}$$

where $C = \text{const}$

Answer: $-\frac{2}{3} \sqrt{1-t}(t+2) + C.$