Conditions

Solve the following system of linear equations by inversion method:
x1 + 6x2 + 4x3 = 2
2x1 + 4x2 - x3 = 3
-x1 + 3x2 + 5x3 =3

Solution

Let's write the system in a matrix form:

$$AX = B: \begin{pmatrix} 1 & 6 & 4 \\ 2 & 4 & -3 \\ -1 & 3 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$$

Let's find the inversion matrix A^{-1} by using cofactors method:

$$A^{-1} = \frac{1}{detA}\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}, A_{ij} = (-1)^{i+j}M_{ij}$$

$$A_{11} = (-1)^2\begin{vmatrix} 4 & -3 \\ 3 & 5 \end{vmatrix} = 20 + 9 = 29$$

…

$$A_{33} = (-1)^6\begin{vmatrix} 1 & 6 \\ 2 & 4 \end{vmatrix} = 4 - 12 = -8$$

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} = \begin{pmatrix} 29 & -18 & -34 \\ -7 & 9 & 11 \\ 10 & -9 & -8 \end{pmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 6 & 4 \\ 2 & 4 & -3 \\ -1 & 3 & 5 \end{vmatrix} = 20 + 9 - 60 + 16 + 18 + 24 = 27$$

$$A^{-1} = \frac{1}{27}\begin{pmatrix} 29 & -18 & -34 \\ -7 & 9 & 11 \\ 10 & -9 & -8 \end{pmatrix}$$

Now let's multiply our matrix equation from the left side by inversion matrix:

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

$$X = \frac{1}{27}\begin{pmatrix} 29 & -18 & -34 \\ -7 & 9 & 11 \\ 10 & -9 & -8 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix} = \frac{1}{27}\begin{pmatrix} 29\cdot 2 - 18\cdot 3 - 34\cdot 3 \\ -7\cdot 2 + 9\cdot 3 + 11\cdot 3 \\ 10\cdot 2 - 9\cdot 3 - 8\cdot 3 \end{pmatrix}$$

$$X = \begin{pmatrix} \frac{-98}{27} \\ \frac{46}{27} \\ \frac{-31}{27} \end{pmatrix}$$