

Write the Taylor polynomial $T_5(x)$ for the function $f(x) = \cos x$ centered at $x = 0$.

Solution:

$$T_5(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{(4)}(0) + \frac{x^5}{5!}f^{(5)}(0)$$

$$f(x) = \cos x$$

$$f(0) = 1$$

$$f'(x) = -\sin x$$

$$f'(0) = 0$$

$$f''(x) = -\cos x$$

$$f''(0) = -1$$

$$f'''(x) = \sin x$$

$$f'''(0) = 0$$

$$f^{(4)}(x) = \cos x$$

$$f^{(4)}(0) = 1$$

$$f^{(5)}(x) = -\sin x$$

$$f^{(5)}(0) = 0$$

$$T_5(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

Answer: $T_5(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$