Conditions

Let the function f(x) be continuous at a point X_o. Assume that f(X_o) > 2. Prove that there exists a neighborhood of X_o such that f(x) > 2 for every x in this neighborhood.

Solution

$\forall \varepsilon > 0\ \exists \delta = \delta(\varepsilon) > 0\ \forall x{:}\ |x - x_0| < \delta\ |f(x) - f(x_0)| < \varepsilon$

$f(x_0) = 2 + k, k > 0$

$|f(x) - f(x_0)| < \varepsilon$

$2 + k - \varepsilon < |f(x)| < 2 + k + \varepsilon$

As we know, these claims are true for all $\varepsilon > 0$. And for those ε, which are:

$0 < \varepsilon < k$

$\exists \delta = \delta(\varepsilon) > 0\ \forall x{:}\ |x - x_0| < \delta\ |f(x) - f(x_0)| < \varepsilon$

And

$2 < 2 + k - \varepsilon < |f(x)| < 2 + k + \varepsilon$

Q.E.D.