Conditions

Prove (using the "epsilon - delta" definition) that the function f(x) = 4x^2 - 5x + 3 is continuous at every point X_0.

Solution

Consider function:

$f(x), x \epsilon [a, b]$

This function is continuous in $x_0 \epsilon [a, b]$, if:

$\forall \varepsilon > 0 \; \exists \delta = \delta(\varepsilon) > 0 \; \forall x: |x - x_0| < \delta \; |f(x) - f(x_0)| < \varepsilon$

Fix $x_0 \epsilon [a, b]$, fix $\varepsilon > 0$

Consider $|f(x) - f(x_0)|$:

$|f(x) - f(x_0)| = |4x^2 - 5x + 3 - 4x_0^2 + 5x_0 - 3| = |4x^2 - 5x - 4x_0^2 + 5x_0| =$

$= |4x^2 - 5x - 4x_0^2 + 5x_0| \leq 4|x - x_0|^2 + 5|x - x_0| \leq 8\delta|b| + 5\delta < \varepsilon$

$\delta < \frac{\varepsilon}{8|b| + 5}$