

Conditions

if $\tan\theta + \sec\theta = x$, prove that $\sin\theta = (x^2 - 1)/(x^2 + 1)$

Solution

We must prove the following:

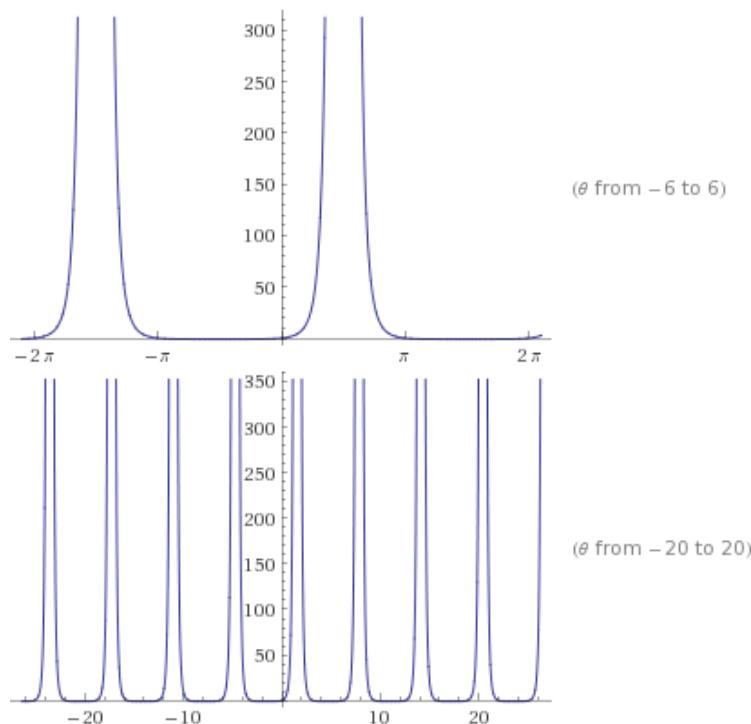
$$\sin\theta = ((\tan\theta + \sec\theta)^2 - 1)((\tan\theta + \sec\theta)^2 + 1)$$

But this is not true for all θ . For example, let's take $\theta = \pi/4$. Then:

$$\begin{aligned} ((\tan\theta + \sec\theta)^2 - 1)((\tan\theta + \sec\theta)^2 + 1) &= ((1 + \sqrt{2})^2 - 1)((1 + \sqrt{2})^2 + 1) \\ &= (1 + \sqrt{2})^4 - 1 \approx 32.97 \end{aligned}$$

And this value must be equal to $\sin\theta$. But sinus is a bounded function in interval $[-1, 1]$. So the equation is wrong.

The graph of $(\tan\theta + \sec\theta)^2 + 1$ is below:



As we see, the graph isn't bounded between -1 and 1 not only in 1 point.