

An investigator thinks that people under the age of forty have vocabularies that are different than those of people over sixty years of age. The investigator administers a vocabulary test to a group of 31 younger subjects and to a group of 31 older subjects. Higher scores reflect better performance. The mean score for younger subjects was 14.0 and the standard deviation of younger subject's scores was 5.0. The mean score for older subjects was 20.0 and the standard deviation of older subject's scores was 6.0. Does this experiment provide evidence for the investigator's theory?

**Solution**

We used formula for the t-test: 
$$= \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$t = \frac{14.0 - 20.0}{\sqrt{\frac{5.0^2}{31} + \frac{6.0^2}{31}}} = -4,28$$

To calculate degrees of freedom we used: 
$$v = \frac{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}\right)^2}{\left(\frac{s_1^2}{N_1}\right)^2 / (N_1 - 1) + \left(\frac{s_2^2}{N_2}\right)^2 / (N_2 - 1)}$$

$$v = \frac{\left(\frac{5.0^2}{31} + \frac{6.0^2}{31}\right)^2}{\frac{(5.0^2/31)^2}{31 - 1} + \frac{(6.0^2/31)^2}{31 - 1}} \cong 58$$

We state the null hypothesis:  $H_0: \mu_1 = \mu_2$

Alternative hypothesis:  $H_a: \mu_1 \neq \mu_2$

Significance level:  $\alpha = 0.05$

Critical region: Reject  $H_0$  if  $|T| > t_{0.975,58}$

**Answer:** For our two-tailed t-test, the critical value is  $t_{0.975,58} = 2.002$ , where  $\alpha = 0.05$  and  $v = 58$ . We reject the null hypotheses for our t-test because the value of the test statistic is higher than the critical value  $t = 4.28 > 2.002$  and investigator is right that people under the age of forty have vocabularies that are different than those of people over sixty years of age.