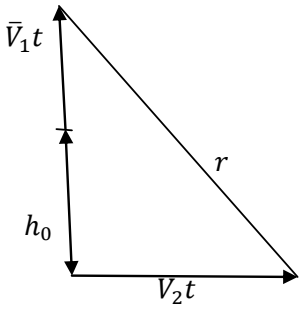
A hot-air balloon is 150 feet above the ground when a motorcycle passes directly beneath it (traveling in a straight line on a horizontal road). The motorcycle is traveling at 40 mph. If the balloon is rising vertically at a rate of 10 ft/sec, what is the rate of change of the distance between the balloon and the motorcycle ten seconds later? (Needed - mathematical model and derivative)

Solution



By the Pythagorean theorem

$$r^2 = (h_0 + V_1 t)^2 + V_2^2 t^2$$

$$r = \sqrt{(h_0 + V_1 t)^2 + V_2^2 t^2}.$$

$$\frac{dr}{dt} = \frac{1}{2}\frac{(2V_2^2 t + 2(h_0 + V_1 t)V_1)}{\sqrt{(h_0 + V_1 t)^2 + V_2^2 t^2}} = \frac{V_2^2 t + V_1(h_0 + V_1 t)}{\sqrt{(h_0 + V_1 t)^2 + V_2^2 t^2}}.$$

$$V_2 = 40\ mph = 17.8816\frac{m}{s}, V_1 = 10\frac{ft}{sec} = 3.048\frac{m}{s}. h_0 = 150\ feet = 45{,}72\ m.$$

$$\frac{dr}{dt}(10\ s) = \frac{17.88^2 * 10 + 3.048 * (45.72 + 3.048 * 10)}{\sqrt{(45{,}72 + 3.048 * 10)^2 + 17.88^2 * 10^2}} = 17{,}64354\frac{m}{s} \cong 17.6\frac{m}{s}$$