## Conditions

Suppose p is a polynomial with n distinct real roots. Show that p' has at least n−1 distinct real roots.

## Solution

Let's use Rolle's Theorem. It claims that for differentiable functions with two points with an equal value, their derivative has a root between these points.

The polynomial is a differentiable function, and as we have n roots, so we have n-1 intervals, where at two distinct points we have equal values (zero, as they are roots).

We have the following n-1 intervals

$(a_1, a_2), (a_2, a_3), \dots (a_{n-1}, a_n)$

$P(a_1) = P(a_2) = \cdots = P(a_n) = 0$

In each interval by Rolle's Theorem we have 1 root for derivative function. Totally – n-1 roots. And the proof is done.

Q.E.D.