Conditions

Diagonalize a 2x2 matrix with values 1, 0, 6, and -1

Solution

$A = \begin{pmatrix} 1 & 0 \\ 6 & -1 \end{pmatrix}$

Let's find the eigenvalues:

$|A - \lambda E| = \begin{vmatrix} 1-\lambda & 0 \\ 6 & -1-\lambda \end{vmatrix} = (1-\lambda)(-1-\lambda) = -1 + \lambda^2$

$\lambda_{1,2} = \pm 1$

Let's find the eigenvectors:

$(A - \lambda E)x = 0$

For λ_1:

$\begin{pmatrix} 0 & 0 \\ 6 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

$\begin{pmatrix} 0 \\ 6x_1 - 2x_2 \end{pmatrix} = 0$

$\begin{pmatrix} x_1 \\ 3x_1 \end{pmatrix}, x_1 \epsilon R$

For λ_2:

$\begin{pmatrix} 2 & 0 \\ 6 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

$\begin{pmatrix} 2x_1 \\ 6x_1 \end{pmatrix} = 0$

$\begin{pmatrix} 0 \\ x_2 \end{pmatrix}, x_2 \epsilon R$

For this vectors let's find a P matrix, for which the diagonal matrix J:

$A = P^{-1} J P$

$J = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$P = \begin{pmatrix} 0 & x_1 \\ x_2 & 3x_1 \end{pmatrix}$

Let's $x_1 = 1$, $x_2 = 1$

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 3 \end{pmatrix}$$

$$\det(P) = -1 \neq 0$$

$$P^{-1} = \frac{1}{\det(P)} C^T$$

Where C – cofactor matrix.

$$P^{-1} = \begin{pmatrix} -3 & 1 \\ 1 & 0 \end{pmatrix}$$