

**Question 18748** Hi, the topic is Probability Density Function of a random variable.

For each of the following, find the constant  $c$  so that  $p(x)$  satisfies the condition of being a probability density function of a random variable  $X$ : I.  $p(x) = c(2/3)^x, x \in N$  II.  $p(x) = cx, x \in \{1, 2, 3, 4, 5, 6\}$

I've been figuring this for couple of days. **Solution.** In fact,  $p(x)$  is rarely called probability density function. It is just distribution of a r.v.,  $p(x) = P(X = x)$ .

We are to verify two conditions: 1)  $p(x) \geq 0$  and  $\sum_{x \in N} p(x) = 1$ . So,

a)  $c > 0$  and  $\sum_{x \in N} p(x) = c \sum_{x \in N} (2/3)^x = c \frac{2/3}{1/3} = 2c = 1$ , thus  $c = 1/2$ . Remark:

this is the case when  $0 \notin N$  (2 definitions of natural numbers are used in mathematics.), if  $0 \in N$ , then  $\sum_{x \in N} p(x) = c \sum_{x \in N} (2/3)^x = 3c$ , thus  $c = 1/3$ .

b)  $\sum_{x \in \{1, 2, 3, 4, 5, 6\}} cx = c \frac{6 \cdot 7}{2} = 21c$ , thus  $c = 1/21$ .