## Conditions

Suppose p is a polynomial with n distict roots. show that p' has n-1 roots.

We have studied MVT and rolles theorem so i would think i need to use them to prove it?

## Solution

Exactly! Here must be used the Rolle's Theorem.
It claims us, that for differentiable function, which have 2 points with equal values on them, exist at least one point between, in which the derivative function's value is 0.

More strict formulation:

If a real-valued function *f* is continuous on a closed interval [*a*, *b*], differentiable on the open interval (*a*, *b*), and *f*(*a*) = *f*(*b*), then there exists a *c* in the open interval (*a*, *b*) such that

$$f'(c) = 0.$$

As we have n roots for our polynomial, and as polynomials in real are differentiable functions, we can say, that we have n values, where function has a 0 value (for us important, that these values are equal).

For n those points we have n-1 intervals, on the edges of which are our roots. Now in each of them let's use the Rolle's Theorem and we will get n-1 zeros for derivative function. And this means that each that point is a root for derivative function.

Q.E.D.