

Question

The price of the materials will be: $p(x, z) = 3x^2 + 24xz$. The volume will be $f(x, z) = x^2z$.

And we know that the price is \$144. So, we have:

$$p(x, z) = 3x^2 + 24xz = 144 \Rightarrow z = \frac{144 - 3x^2}{24x} = \frac{6}{x} - \frac{x}{8}.$$

The volume will be in this case:

$$f(x) = x^2 \cdot \left(\frac{6}{x} - \frac{x}{8} \right) = 6x - \frac{x^3}{8}.$$

Answer: the volume is $f(x) = 6x - \frac{x^3}{8}$.

We can find maximum volume:

$$f'(x) = 6 - \frac{3x^2}{8} = 0 \Rightarrow x = 4. \text{ So, the base has side 4 meters in length. Then height of the box}$$

will be: $z = \frac{6}{4} - \frac{4}{8} = \frac{3}{2} - \frac{1}{2} = 1$. So, the dimensions are: $x = 4$ and $z = 1$.

Answer: the dimensions are: $x = 4$ and $z = 1$.