# Conditions

find the eigenvalue and eigenvector for matrix

$$S= \begin{matrix} 0 & 0 & 5 \\ -2 & 1 & 1 \\ 3 & 0 & 2 \end{matrix}$$

Please show your work

# Solution

$$S = \begin{pmatrix} 0 & 0 & 5 \\ -2 & 1 & 1 \\ 3 & 0 & 2 \end{pmatrix}$$

The definition of an eigenvalue claims, that they are values of $\lambda$, which could be found by solving the following matrix equation:

$$\det(S - \lambda E) = 0$$

$$|S - \lambda E| = \begin{vmatrix} -\lambda & 0 & 5 \\ -2 & 1-\lambda & 1 \\ 3 & 0 & 2-\lambda \end{vmatrix} = -\lambda \begin{vmatrix} 1-\lambda & 1 \\ 0 & 2-\lambda \end{vmatrix} - 0 + 5 \begin{vmatrix} -2 & 1-\lambda \\ 3 & 0 \end{vmatrix}$$

$$= -\lambda(1-\lambda)(2-\lambda) - 0 - 0 + 0 - 15(1-\lambda) = \big((-\lambda)(2-\lambda) - 15\big)(1-\lambda)$$

$$= 0$$

$$\lambda_1 = 1$$

$$(-\lambda)(2-\lambda) - 15 = 0$$

$$\lambda^2 - 2\lambda - 15 = 0$$

$$\lambda_2 = -3$$

$$\lambda_3 = 5$$

**Answer: The eigenvalues are:**

$$\lambda_1 = 1$$

$$\lambda_2 = -3$$

$$\lambda_3 = 5$$